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On a Sustainability Interval Index and Its Computation Through Global Optimization

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Prior sustainability models have taken exact basic indicator (BI) values of an entity's sustainability and computed an exact value of the entity's sustainability index. However, BI values are either uncertain or difficult to obtain. In this work, we propose a novel approach that transforms BI data in the form of intervals containing all possible BI values to an interval containing all possible values of the sustainability index. This interval is termed the ''sustainability index interval'' (SII). Computation of the SII is achieved through solution of a minimization and a maximization problem using global optimization techniques. Although the underlying global optimization problems are nonconvex, they are shown to possess a number of properties that can be utilized to reduce the burden associated with SII's computation. Based on these properties, a branch-and-bound algorithm is developed, which exactly quantifies the SII in a finite number of iterations. *AIChE J,* 58: 2743–2757, 2012
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Introduction

From the middle of the 20th century, as the human impact on the environment became apparent, legal frameworks for environmental protection began to be implemented. This led to ''end-of-pipe'' treatment strategies intent on minimizing pollutant emissions and the development of industrial waste minimization and pollution prevention strategies (recycling, reuse, waste reduction, etc.). The limitations of these strategies led to the proposition that process synthesis techniques be used for creating facilities with minimal waste generation.[1,2]

The realization that environmental problems often span multiple areas and time frames led to the concept of sustainability. Defining ''sustainability'' is a difficult task; multiple definitions of sustainability exist. The United Nations Environment Program Brundtland report describes ''sustainable development'' as ''[meeting] the needs of the present without compromising the ability of future generations to meet their own needs.''[3] It outlines three main aspects of sustainability: ecological impact, economic development, and societal equity.[3] An attempt at a sustainability definition from professionals at the U.S. EPA's National Risk Management Research Laboratory suggests that ''sustainability occurs when we maintain or improve the material and social conditions for human health and the environment over time without exceeding the ecological capabilities that support them.''[4] Cabezas and Fath used Fisher information theory to quantify sustainability, proposing that ''sustainable systems do not lose or gain Fisher information over time.''[5] AIChE's Institute for Sustainability defines sustainability as ''the path of continuous improvement, wherein the products and services required by society are delivered with progressively less negative impacts upon the earth.''[6]

Previous attempts to quantify sustainability include the ecological footprint,[7,8] the use of Fisher information theory,[5] the pressure-state-response model of environmental sustainability,[9] 3-D metrics quantifying the main aspects of sustainability outlined in the Brundtland report,[4] the environmental sustainability index,[10] the sustainability assessment by fuzzy evaluation (SAFE) model,[11–14] and the AIChE sustainability index.[6,15] SAFE has been used to determine the sustainability of nations and corporations.[11,14] In this work, ''sustainability index'' refers to the index for the SAFE model.

The main contribution of this article is that it extends the results of Kouloumpis et al.[11] to cases where an additional source of uncertainty is present, that of the input data values. All inputs are assumed known in Ref. [11] and given by crisp numbers. Here, only intervals of input values are known, not single values. This is quite natural given that indicator

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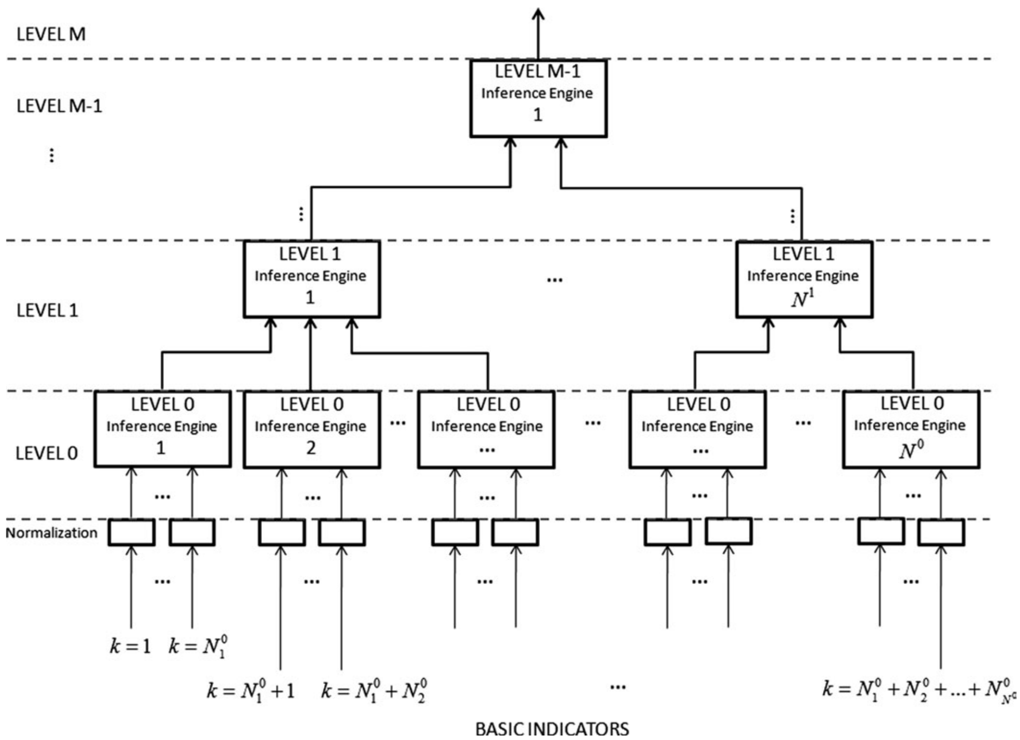

Figure 1. Multilevel hierarchical structure for quantifying sustainability.

statistics often vary according to source. Take for example the GINI coefficient, which measures how equitably the national product of a country is distributed among its citizens. The UN and the CIA provide tables for this indicator for most member states of the UN, but their numerical values often differ. The same is true if one looks at statistics by different sources, say the World Bank and World Resources Institute. Data sometimes are provided by organizations in terms of intervals because of uncertainty. For example, in the work of Desai et al.,[16] solid fuel use and related respiratory risk are given via low and high estimates depending on the factors that are taken into account. Such factors are dominant fuel used for cooking (dung, charcoal, wood, or crop residues), stove type, kitchen location, etc. When value intervals are considered the problem is reduced to the computation of an interval for the sustainability index, called the sustainability index interval (SII). SII differs from other indices such as Eco-indicator 99, CML 2001, or the IPCC characterization for the global warming potential of airborne emissions. All these indices are primarily used at the impact evaluation stage of life cycle assessment (LCA). Eco-indicator 99 values concern materials, production processes, transport processes, energy generation, and disposal of materials.[17] Its purpose is to come up with an assessment of impact expressed as an impact score. Similarly, CML-2001[18] is focused on LCA. SII is more general and global as it assesses the sustainability standing of an industry or a whole country and can incorporate such disparate inputs as biodiversity, emissions of pollutants, human rights, education, and the economy. Its goal is two-fold: First, rigorously compute a sustainability interval; second, pinpoint those indicators that affect SII the most. These are the indicators where decision makers should focus primarily. SII has the advantage of guaranteeing global optimality of its solutions because it does not rely on randomized data inputs as in Monte Carlo methods.

In the ''Conceptual Framework'' section, the properties of the underlying global optimization problems will be established. Through mathematical proof, the SII's guarantee in identifying the global optima for these problems will be verified. These properties will then be utilized in the development of a finite algorithm that exactly computes the tightest possible interval of the SII for an arbitrary collection of initial intervals. The ''Case Study'' section, through two examples, will illustrate the power of the proposed algorithm in quantifying SII for a corporation and detail potential decision-making strategies utilizing the interval edges of the SII.

Conceptual Framework

The goal in developing SII for an entity is to create a comparative tool that will allow quantification of sustainability analysis, so it can be scientifically reproducible and can be used to rank the entity among its peers in relation to its sustainability status. This entails first identifying, for the entity under consideration, factors that can be used to assess its sustainability efforts. This activity is carried out in a multilevel hierarchical manner, as pictured in Figure 1 below. At the highest level (level M) we identify broad areas (factors) that will influence our assessment of an entity's sustainability. For example, in the context of the Brundtland report,[3] three broad factors are considered (ecological impact, economic development, and societal equity). As these broad factors are analyzed, their constitutive components are then identified, and then their components' components, and so on. As information granularity increases, the hierarchical level decreases to "level M-1," "level M-2," ..., until "level 0." The factors at the level with the highest information granularity (i.e., level 0) are termed the "basic indicators" (BIs) of sustainability. A BI can be a measurable quantity such as "CO_2 emissions per capita" or a value obtained from subjective assessment such as "compliance with environmental laws."[11]

The diverse nature of the various BIs of sustainability necessitates the development of a procedure that normalizes these indicators, so they can assume values in similar ranges. These "normalized BIs" (NBIs) can then be combined in an appropriate manner to give rise to a sustainability index. Given our desire to develop a sustainability index that can serve as a comparative tool for the ranking of various entities, it is natural to use in the normalization process the BI values attained by all considered entities. By defining a normalization function (typically piecewise linear) we are able to create "NBIs," which assume values in the interval [0,1]. Appropriate selection of the normalization function allows us to incorporate our intuitive preferences on how BI values should compare with each other. For example, for a BI function such as "CO_2 emissions per capita," the normalization function may be chosen to be linear for all values between the lowest and highest values of this indicator among all entities and to then level off at 0 (1) for BI values above (below) the aforementioned highest (lowest) BI values. This normalization function selection reflects a desire to not reward or penalize BI values beyond certain threshold values.

Upon completion of the normalization process it is desired that the "NBIs" be utilized to provide an assessment of an entity's sustainability status. There are several challenges associated with the accomplishment of this goal. First, "BI" information may be uncertain or even unavailable. Second, sustainability assessment is fundamentally a subjective process, wherein if two individuals are presented with the same information regarding an entity's sustainability one may characterize it as "weak" while another may characterize it as "medium." Finally, the creation of an overall sustainability index requires the progressive aggregation of "BI" information into higher and higher level "composite indicator" information that in turn creates a hierarchy tree (see Figure 1) at whose top the overall sustainability index is evaluated.

In this work, addressing the BI uncertainty/unavailability challenge will be pursued through the introduction of an interval for each BI within which all values of that BI are guaranteed to belong. Addressing the subjectivity and aggregation challenges will be pursued through modification of the SAFE model,[11–14] which uses fuzzy set theory to address subjectivity[19] so that it allows the use of interval valued "BIs." Linguistic variable values will be assigned to BI values to address subjectivity, and rule sets assigning higher level linguistic variable values to lower level linguistic variable values will be used to create the hierarchy tree (see Figure 1) that will allow computation of the SII.

There are a number of reasons why fuzzy assessment seems to be a natural way of assessing sustainability. Two such reasons follow:

1. Sustainability is an inherently vague and complex concept and cannot be described, let alone measured, by traditional mathematics easily. Decision makers and politicians often prefer to use related natural language expressions rather than numbers or equations. Take, for example, the sustainability indicators "law enforcement," "civil liberties," or "human rights" in a country. They are usually obtained linguistically via subjective assessments of the type "bad," "average," or "good," which fit precisely the settings of linguistic variables and fuzzy sets.

2. To model a system whose structure is not known well, often statistics and system identification are used. Such models require a number of input–output measurements, a collection of candidate models, and a criterion to select the best model that fits these measurements. To assess sustainability using these methods one has to understand and estimate the output data. Although many BIs (inputs) are measurable, it is very difficult to estimate outputs, e.g., human system sustainability or overall sustainability.

Fuzzy logic is suitable for assessing sustainability because it can model complex systems fraught with subjectivity, whose dynamics are insufficiently known. In addition, fuzzy logic can handle knowledge and data represented in various ways such as mathematical models, linguistic rules, numerical values, or linguistic expressions, which is precisely the case of sustainability indicators.

It should be noted that most established models of sustainability also use subjective assessments albeit not as involved as fuzzy logic. Two such models are as follows[20]:

The barometer of sustainability

The International Union for the Conservation of Nature has developed the barometer of sustainability, which has two fundamental components, ecosystem well-being and human well-being. Assessments are given by subjectively defining five bands for each indicator on a scale [0,100]:

Bad (0–20, unsustainable), poor (21–40, almost unsustainable), medium (41–60), OK (61–80, almost sustainable), and good (81–100, sustainable).

Sustainable society index

The sustainable society index (SSI) uses 22 indicators grouped into five main categories: personal development, clean environment, sustainable use of resources, well-balanced society, sustainable use of resources, and sustainable world. Aggregation is again performed via a subjective-weighted average to provide a sustainability index and the corresponding ranking. For more details, see Ref. 20 and relevant references cited.

The first step in presenting the proposed conceptual framework is to provide a brief introduction to fuzzy sets:

A fuzzy set A is defined to be an ordered pair of a set X and a function μ_A: $X \to [0,1]$, i.e., $A = (X, \mu_A)$. In essence, A can be thought of as the graph of μ_A: $A = \{(x, \mu_A(x)) \in X \times [0, 1]\}$. The symbol μ_A is termed the membership grade function, and the value $\mu_A(x)$ is the membership grade of x to A $\forall\ x \in X$ and denotes the degree to which we consider the statement $\{x$ belongs to $A\}$ to be true. When μ_A can take only values of 0 and 1, with $\mu_A(x) = 1(0)$ indicating x does (does not) belong to A, then $\mu_A(x)$ is simply the familiar characteristic function of the classical set, and A is considered a set in the ordinary sense of the term.[19] The support of a fuzzy set A is a subset of the set X defined as follows: support$(A) = \{x \in X : \mu_A(x) > 0\}$. The height and peak values of A are as follows: $h_A = \text{height}(A) = \max_{x\in X} \mu_A(x)$ and $Y_A = \text{peak value}(A) = \arg\max_{x\in X} \mu_A(x)$.

Let B and C be fuzzy sets. Then, the following properties can be either found in Zadeh[19] or be easily verified to hold:

$$
\begin{aligned}
&A = (X, \mu_A) \wedge B = (X, \mu_B) \wedge C = (X, \mu_C) \Rightarrow \\
&\left[\begin{array}{ll}
A = B \Leftrightarrow \mu_A(x) = \mu_B(x) & \forall x \in X \text{ (equality)} \\
A = \varnothing \Leftrightarrow \mu_A(x) = \mu_\varnothing(x) = 0 & \forall x \in X \text{ (emptyset)} \\
A \subset B \Leftrightarrow \mu_A(x) \le \mu_B(x) & \forall x \in X \text{ (subset)} \\
C = A \cap B \Rightarrow \mu_C(x) \le \min(\mu_A(x), \mu_B(x)) & \forall x \in X \text{ (intersection)} \\
C = A \cup B \Rightarrow \mu_C(x) \ge \max(\mu_A(x), \mu_B(x)) & \forall x \in X \text{ (union)} \\
A \cap B = \varnothing \Rightarrow \langle C = A \cup B \Leftrightarrow \mu_C(x) = \mu_A(x) + \mu_B(x) \quad \forall x \in X \rangle & \text{(disjoint union)}
\end{array}\right] \\
&A = (X_1, \mu_A) \wedge B = (X_2, \mu_B) \wedge C = (X_1 \times X_2, \mu_C) \Rightarrow \\
&\quad \langle C = A \times B \Leftrightarrow \mu_C((x_1, x_2)) = \mu_A(x_1) \cdot \mu_B(x_2) \rangle \ \forall (x_1, x_2) \in X_1 \times X_2 \text{ (Cartesian product).}
\end{aligned}
$$

The practical meaning of the fuzzy set A is that, for each element $x \in X$, the degree to which the statement $x \in A$ is true is equal to the membership grade value $\mu_A(x)$. In the remaining of this work, and in the interest of simplicity albeit at the expense of some rigor, the symbol A will also be referred to as a linguistic variable whose values corresponding to the BI level are W, M, S. We consider that membership grade functions can be determined using surveys. To directly correlate survey results with membership grade functions, we consider that surveys are structured so that every participant has an opinion, and only one opinion, about how a BI value compares to other values of the same BI. In turn, this leads to fuzzy sets that are disjoint and allows for the exact computation of fuzzy set unions.

To begin developing the conceptual framework for SII calculation, we consider that the BIs ($x_1, x_2, \ldots, x_n$) of some entity (e.g., a nation and corporation) are each within an interval range $[x_k^L, x_k^U]$, $1 \le k \le n$. As these BIs, in general, do not use the same units, they must first be normalized to the same scale. The normalization procedure from the SAFE model[14] is initiated by transforming the BIs $\{x_k\}_{k=1}^n$ to the level 0 NBI values $\{y_k{}^0\}_{k=1}^n$ using trapezoidal functions. Figures 2a–c illustrate the three types of normalization considered in this work. T_k and t_k are the maximum and minimum, respectively, of the BI value x_k for which the level 0 NBI value y_k^0 is 1. U_k and u_k are the maximum and minimum values, respectively, of x_k for which y_k^0 is nonzero. M_k and m_k are the BI values in $[T_k, U_k]$ and $[u_k, t_k]$, respectively, for which $y_k^0 = \alpha_k$, a threshold value which will later be used to define the range of level 0 NBI values where various membership grade functions assume their maximum or minimum values (see Figure 3 below).

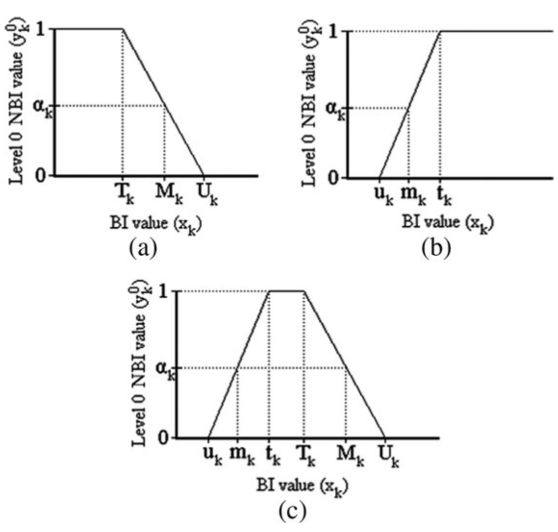

Figure 2. General forms of the BI normalization function.

Equation 1 quantifies the normalization procedure illustrated in Figure 2c, which stipulates that an interval $[t_k, T_k]$ is considered fully sustainable ($y_k^0 = 1$).

$$
y_k^0(x_k) = \begin{cases}
(0)x_k + (0), & x_k \le u_k \\
\left(\frac{1}{t_k - u_k}\right)x_k - \left(\frac{u_k}{t_k - u_k}\right), & u_k \le x_k \le t_k \\
(0)x_k + (1), & t_k \le x_k \le T_k \\
-\left(\frac{1}{U_k - T_k}\right)x_k + \left(\frac{U_k}{U_k - T_k}\right), & T_k \le x_k \le U_k \\
(0)x_k + (0), & U_k \le x_k
\end{cases} \tag{1}
$$

Clearly, the function $y_k^0(\cdot)$ is piecewise linear in the domain of x_k. If, for some k, any level 0 BI value below the target value T_k is deemed fully sustainable, the normalization function will have the form shown in Figure 2a. If any level 0 BI value above the target value t_k is considered fully sustainable, Figure 2b describes the form of the normalization function. Analogous equations corresponding to Figures 2a, b for some k can be obtained from Eq. 1 by setting u_k, $t_k = 0$ and T_k, $U_k = \infty$, respectively.

Once the level 0 normalization process is completed, level 0 fuzzy sets (and their associated linguistic variables) are used to capture the range of survey opinions that may be expressed regarding the sustainable nature of each level 0

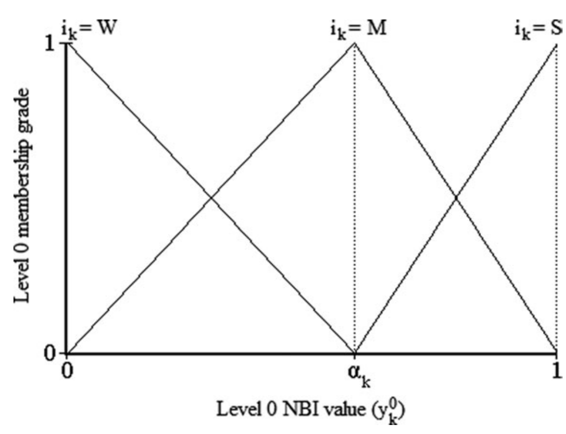

Figure 3. Membership grade functions for the BIs.

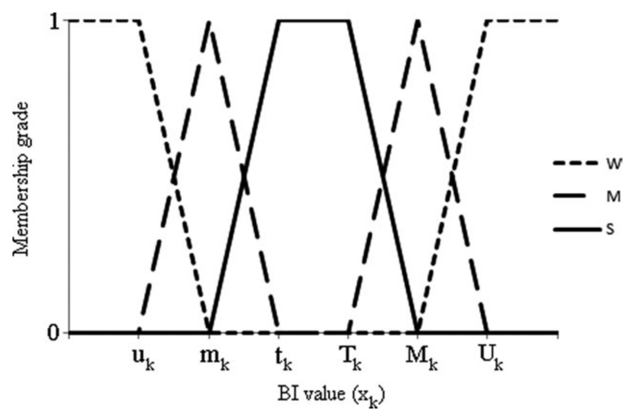

Figure 4. Level 0 membership grades (middle value desired).

NBI of the entity under consideration. Let k denote the level 0 NBI index, I_k^0 denote the collection of level 0 fuzzy sets for indicator k, $i_k \in I_k^0$ denote a fuzzy set for indicator k, and $\mu_{k,i_k}^0(y_k^0)$ denote the value of the membership grade function of the level 0 NBI y_k^0 to the fuzzy set $i_k \in I_k^0$.[21] In this work, we consider that $I_k^0 = \{W, M, S\}\ \forall\ k \in \{1, 2,\ldots, n\}$, i.e., $i_k = W$, $i_k = M$, $i_k = S$ indicate the fuzzy sets corresponding to linguistic variable values "weak," "medium," and "strong," respectively, for any indicator k. We then define the membership grade functions for the level 0 fuzzy sets so that their values $\mu_{k,i_k}^0(y_k^0)$ are given by Eqs. 2–4 and illustrate them in Figure 3. They are essentially triangular functions that satisfy $\mu_{k,W}^0(0) = 1$, $\mu_{k,M}^0(0) = 0$, $\mu_{k,s}^0(0) = 0$, $\mu_{k,W}^0(\alpha_k) = 0$, $\mu_{k,M}^0(\alpha_k) = 1$, $\mu_{k,S}^0(\alpha_k) = 0$, and $\mu_{k,W}^0(1) = 0$, $\mu_{k,M}^0(1) = 0$, $\mu_{k,S}^0(1) = 1$.

$$\mu_{k,W}^0(y_k^0) = \begin{cases} -\left(\frac{1}{\alpha_k}\right)y_k^0 + (1), & 0 \le y_k^0 \le \alpha_k \\ (0)y_k^0 + (0), & \alpha_k \le y_k^0 \le 1 \end{cases} \tag{2}$$

$$\mu_{k,M}^0(y_k^0) = \begin{cases} \left(\frac{1}{\alpha_k}\right)y_k^0 + (0), & 0 \le y_k^0 \le \alpha_k \\ -\left(\frac{1}{1-\alpha_k}\right)y_k^0 + \left(\frac{1}{1-\alpha_k}\right), & \alpha_k \le y_k^0 \le 1 \end{cases} \tag{3}$$

$$\mu_{k,S}^0(y_k^0) = \begin{cases} (0)y_k^0 + (0), & 0 \le y_k^0 \le \alpha_k \\ \left(\frac{1}{1-\alpha_k}\right)y_k^0 - \left(\frac{\alpha_k}{1-\alpha_k}\right), & \alpha_k \le y_k^0 \le 1 \end{cases} \tag{4}$$

Using Eq. 1 to substitute for y_k^0 in Eqs. 2–4 in terms of x_k permits the explicit evaluation of the composite membership function with values $\mu_{k,i_k}^0(x_k)$. As both the functions $\mu_{k,i_k}^0(y_k^0)$ and $y_k^0(x_k)$ are piecewise linear, it is easy to establish that the composite membership function $\mu_{k,i_k}^0(x_k)$ is also piecewise linear. Defining appropriate subintervals and using Eqs. 1–4 leads to the composite function expressions below (Eqs. 5–7) corresponding to Figure 4. As before, setting u_k, m_k, $t_k = 0$ and T_k, M_k, $U_k = \infty$ yields analogous composite membership functions for Figures 5 and 6, respectively.

$$\mu_{k,W}^0(x_k) = \left\{ \begin{array}{l} (0)x_k + (1),\ x_k \in (-\infty, u_k] \\ -\left(\frac{1}{\alpha_k(t_k-u_k)}\right)x_k + \left(\frac{(1-\alpha_k)u_k+\alpha_k t_k}{\alpha_k(t_k-u_k)}\right),\ x_k \in [u_k, m_k] \\ (0)x_k + (0),\ x_k \in [m_k, t_k] \\ (0)x_k + (0),\ x_k \in [t_k, T_k] \\ (0)x_k + (0),\ x_k \in [T_k, M_k] \\ \left(\frac{1}{\alpha_k(U_k-T_k)}\right)x_k - \left(\frac{(1-\alpha_k)U_k+\alpha_k T_k}{\alpha_k(U_k-T_k)}\right), x_k \in [M_k, U_k] \\ (0)x_k + (1),\ x_k \in [U_k, \infty) \end{array} \right\} \tag{5}$$

$$\mu_{k,M}^0(x_k) = \left\{ \begin{array}{l} (0)x_k + (0),\ x_k \in (-\infty, u_k] \\ \left(\frac{1}{\alpha_k(t_k-u_k)}\right)x_k - \left(\frac{u_k}{\alpha_k(t_k-u_k)}\right),\ x_k \in [u_k, m_k] \\ -\left(\frac{1}{(1-\alpha_k)(t_k-u_k)}\right)x_k + \left(\frac{t_k}{(1-\alpha_k)(t_k-u_k)}\right),\ x_k \in [m_k, t_k] \\ (0)x_k + (0),\ x_k \in [t_k, T_k] \\ \left(\frac{1}{(1-\alpha_k)(U_k-T_k)}\right)x_k - \left(\frac{T_k}{(1-\alpha_k)(U_k-T_k)}\right),\ x_k \in [T_k, M_k] \\ -\left(\frac{1}{\alpha_k(U_k-T_k)}\right)x_k + \left(\frac{U_k}{\alpha_k(U_k-T_k)}\right), x_k \in [M_k, U_k] \\ (0)x_k + (0),\ x_k \in [U_k, \infty) \end{array} \right\} \tag{6}$$

$$\mu_{k,S}^0(x_k) = \left\{ \begin{array}{l} (0)x_k + (0),\ x_k \in (-\infty, u_k] \\ (0)x_k + (0),\ x_k \in [u_k, m_k] \\ \left(\frac{1}{(1-\alpha_k)(t_k-u_k)}\right)x_k - \left(\frac{(1-\alpha_k)u_k+\alpha_k t_k}{(1-\alpha_k)(t_k-u_k)}\right),\ x_k \in [m_k, t_k] \\ (0)x_k + (1),\ x_k \in [t_k, T_k] \\ -\left(\frac{1}{(1-\alpha_k)(U_k-T_k)}\right)x_k + \left(\frac{(1-\alpha_k)U_k+\alpha_k T_k}{(1-\alpha_k)(U_k-T_k)}\right),\ x_k \in [T_k, M_k] \\ (0)x_k + (0), x_k \in [M_k, U_k] \\ (0)x_k + (0),\ x_k \in [U_k, \infty) \end{array} \right\} \tag{7}$$

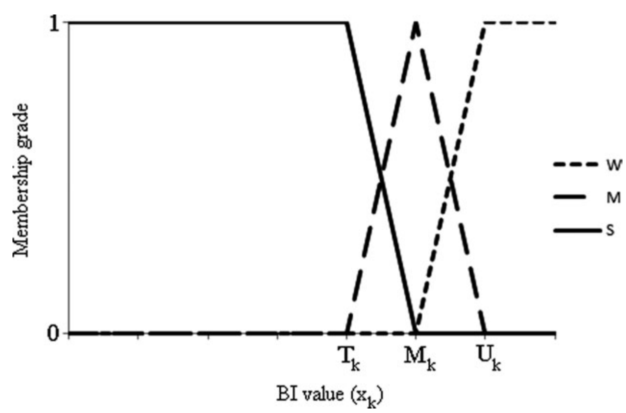

Figure 5. Level 0 membership grades (lower value desired).

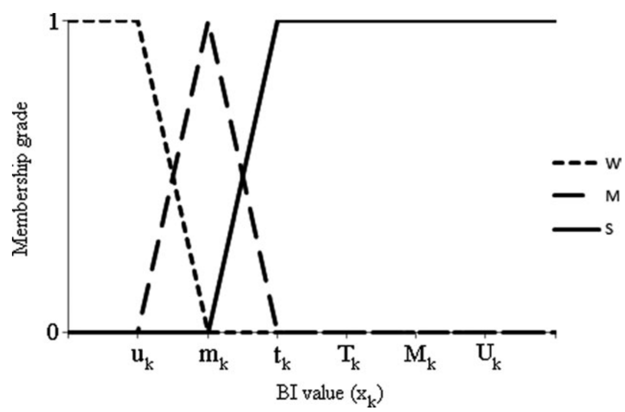

Figure 6. Level 0 membership grades (higher value desired).

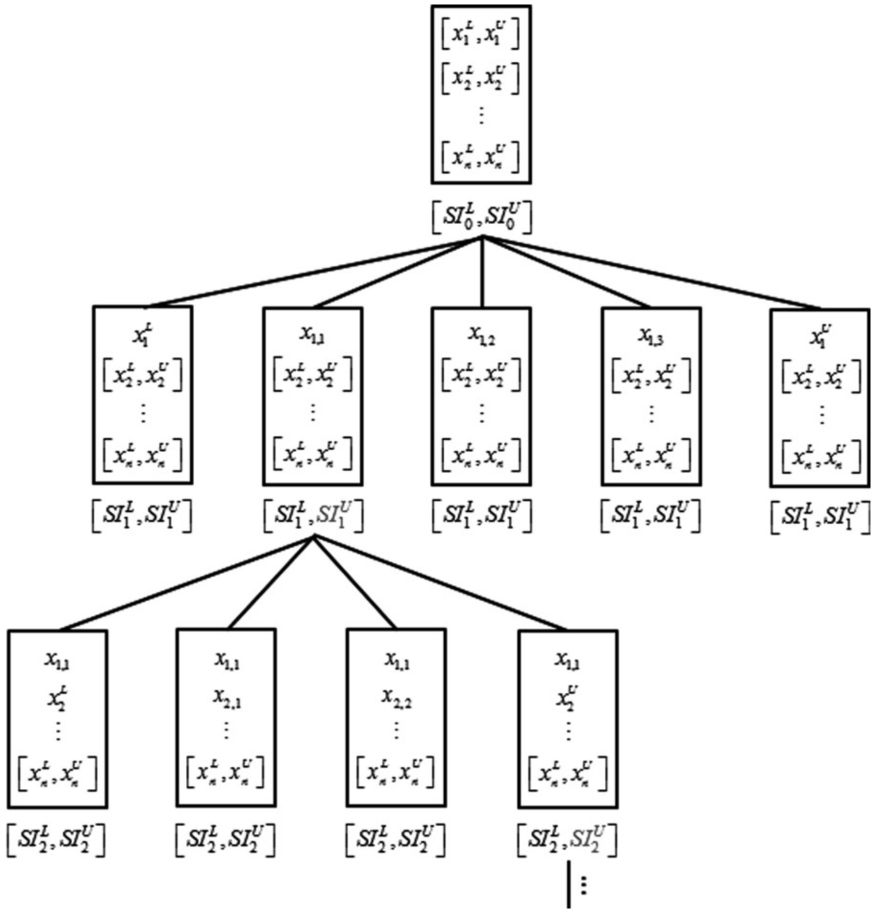

Figure 7. “Branching” optimization algorithm diagram.

To progress to the next level of hierarchy, we first consider that a set of inference engines are defined, which map each level (say level m) to the next level above (level $m + 1$). Figure 1 illustrates the inference engines (shown as boxes) that translate level m fuzzy sets and associated membership grades to level $m + 1$ fuzzy sets and membership grades for level $m + 1$ using various rules and mathematical expressions to be defined. The inputs of each inference engine that maps level 0 to level 1 are some of the considered BIs. The output of this inference engine is a composite indicator to which some physical meaning can be assigned. For example, in the case study considered later in this manuscript, ‘‘greenhouse gas emissions’’ and ‘‘toxic gas emissions’’ are considered as level 0 BIs, which are inputs of an inference engine, whose output is the level 1 composite indicator ‘‘air sustainability.’’

Each one of these inference engines has a set of inputs and one output. Let P^m be the index set of level m inference engines mapping m to $m + 1$; P^m_j be the index set of inputs to the level m inference engine j; N^m denote the cardinality of the set P^m; and N^m_j denote the cardinality of the set P^m_j. Level m fuzzy sets (and their associated linguistic variables) are then used to capture the range of survey opinions that may be expressed regarding the sustainable nature of each level m composite indicator. Similarly to the level 0 case, let k denote a level m composite indicator index, I^m_k denote the collection of level m fuzzy sets for indicator k, and $i_k \in I^m_k$ denote a fuzzy set (or its linguistic variable) for a level m indicator k. However for $m \geq 1$, although the membership grade function μ^m_{k,i_k} associated with the level m indicator k and the fuzzy set i_k can be defined in terms of a normalized composite indicator y^m_k whose domain is the interval [0, 1], it cannot be evaluated based on the above definition, as any level m indicator k is the output of some level $m - 1$ inference engine $j(k \in N^{m-1})$ and its values cannot be derived from the indicators that are inputs to this engine. On the other hand, peak values for the fuzzy sets associated with each such composite indicator are well defined (as an integral part of the definition of the membership functions for these fuzzy sets), and those associated with the level M composite indicators termed ‘‘overall sustainability’’ are used as weights in the definition of the sustainability index.

The values of the membership grade function μ^m_{k,i_k} can be computed by utilizing the assignment of inputs and outputs to each inference engine at every level. Let R^m_{j,i_j} be the set of all $(i_p)_{p \in P^m_j}$ mapping to the same fuzzy set $i_j \in I^{m+1}_j$. Then, Eq. 8 determines the membership grades for level $m + 1$, indicator j, fuzzy set i_j

$$\mu^{m+1}_{j,i_j} = \sum_{(i_p)_{p \in P^m_j} \in R^m_{j,i_j}} \prod_{p \in P^m_j} \mu^m_{p,i_p} \quad m \in \{0, 1, ..., M-1\}. \tag{8}$$

A variety of rule sets that convert level m fuzzy sets to level $m + 1$ fuzzy sets, where $m \in \{0,1,\ldots,M - 1\}$ can be used, as discussed in Phillis and Davis.[14]

Once the membership grades of level M are obtained, the overall sustainability output value SI can be determined by height defuzzification[14,21]

$$\mathrm{SI} = \frac{\sum_{i_1 \in I_1^M} \mu_{1,i_1}^M \cdot Y_{i_1}^M}{\sum_{i_1 \in I_1^M} \mu_{1,i_1}^M} \tag{9}$$

where $Y_{i_1}^M$ is the peak value for the level M fuzzy set i_1 at which the membership function μ_{1,i_1}^M is maximized. Because of how the membership functions are defined, it holds that $\sum_{i_1 \in I_1^M} \mu_{1,i_1}^M = 1$, and since $Y_{i_1}^M \in [0,1]$, it follows that SI $\in$ [0,1]. Finding the interval for SI as the BIs vary involves solving the following minimization and maximization global optimization problems

$$\begin{aligned} &\min, \max \sum_{i_1 \in I_1^M} \mu_{1,i_1}^M \cdot Y_{i_1}^M \\ &\text{s.t.} \\ &\mu_{j,i_j}^{m+1} = \sum_{(i_p)_{p \in P_j^m} \in R_{j,i_j}^m} \prod_{p \in P_j^m} \mu_{p,i_p}^m, \quad m \in \{0,1,\dots,M-1\} \\ &\mu_{k,i_k}^0 = \mu_{k,i_k}^0(x_k), \quad i_k \in \{W,M,S\}, \quad k \in \{1,2,\dots,n\} \\ &x_k^L \le x_k \le x_k^U, \quad k \in \{1,2,\dots,n\} \end{aligned} \tag{10}$$

Based on this problem statement, if only the SII of the topmost level M is desired, then the intermediate membership grade functions $\mu_{j,i_j}^m(y_k^m)$, $m \in \{1,2,\dots,M-1\}$ are unnecessary because the objective function includes only the peak values for the topmost level and terms that are essentially functions of only level 0 membership grades. In essence, the role of the intermediate levels is captured entirely in Eq. 8. However, one may choose to compute intermediate SIIs for various composite indicators as information aggregation proceeds to obtain a more thorough picture of where an entity's sustainability stands. The mathematical formulation of the problem remains unchanged, and simply a portion of the hierarchy tree is ignored. For these subproblems, if the topmost level is m^* then it is necessary to define membership grade functions $\mu_{j,i_j}^{m^*}(y_k^{m^*})$, so that the level m^* peak values are known for the new objective function.

Theorem. To obtain the global optimum of optimization problem (10), it is sufficient to inspect only the "breakpoints" of the objective function, i.e., at the optimum $x_k \in \{x_k^L, u_k, m_k, t_k, T_k, M_k, U_k, x_k^U\}$, $k \in \{1,2,\dots,n\}$.

Proof. Suppose that some BI x_{k^*}, $k^* \in \{1,2,\dots,n\}$ is defined in $[x_{k^*}^L, x_{k^*}^U]$, and that all other BIs x_k, $k \neq k^*$ are fixed but unknown in $[x_k^L, x_k^U]$, $k \neq k^*$. Then, the maximization objective function of Eq. 10 can be written as

$$\max_{\substack{x_k^L \le x_k \le x_k^U \\ k \in \{1,2,\dots,n\}}} \sum_{i_1 \in I_1^M} \mu_{1,i_1}^M \cdot Y_{i_1}^M \Leftrightarrow \max_{\substack{x_k^L \le x_k \le x_k^U \\ k \neq k^*}} \left[\max_{x_{k^*}^L \le x_{k^*} \le x_{k^*}^U} \sum_{i_1 \in I_1^M} \mu_{1,i_1}^M \cdot Y_{i_1}^M \right]$$

As all level $m+1$ membership grades can be obtained from the sum of all possible products of the level m membership grades for $m \in \{0,1,\dots,M-1\}$, μ_{1,i_1}^M can be rewritten as the sum of all possible products of the level 0 membership grades. That is

$$\max_{\substack{x_k^L \le x_k \le x_k^U \\ k \neq k^*}} \left[\max_{x_{k^*}^L \le x_{k^*} \le x_{k^*}^U} \sum_{i_1 \in I_1^M} Y_{i_1}^M \cdot \left(\sum \prod \left\{ \mu_{k,i_k}^0 \right\}_{k=1}^n \right) \right], \; i_k \in I_k^0 = \{W,M,S\}$$

$$\Rightarrow \max_{\substack{x_k^L \le x_k \le x_k^U \\ k \neq k^*}} \left[\max_{x_{k^*}^L \le x_{k^*} \le x_{k^*}^U} \sum_{i_1 \in I_1^M} Y_{i_1}^M \cdot \left(\sum \left(\mu_{k^*,i_{k^*}}^0 \right) \prod \left\{ \mu_{k,i_k}^0 \right\}_{\substack{k=1 \\ k \neq k^*}}^n \right) \right], \; i_k \in \{W,M,S\}$$

From Eqs. 5–7, we know that $\mu_{k^*,i_{k^*}}^0(x_{k^*})$ is linear over certain subintervals $S_{k^*}^\ell$, so our objective function entails maximizing $\mu_{k^*,i_{k^*}}^0$ over each subinterval, then taking the highest maximum from the set of maxima obtained. We now have

$$\max_{\substack{x_k^L \le x_k \le x_k^U \\ k \neq k^*}} \left[\max_{\ell} \left[\max_{x_{k^*} \in S_{k^*}^\ell} \sum_{i_1 \in I_1^M} Y_{i_1}^M \cdot \left(\sum \left(A_{k^*}^\ell \cdot x_{k^*} + B_{k^*}^\ell \right) \prod \left\{ \mu_{k,i_k}^0 \right\}_{\substack{k=1 \\ k \neq k^*}}^n \right) \right] \right], \; i_k \in \{W,M,S\}$$

where $A_{k^*}^\ell$ and $B_{k^*}^\ell$ are constants. But the product of all $\left\{ \mu_{k,i}^0 \right\}_{\substack{k=1 \\ k \neq k^*}}^n$ terms is a fixed value as it depends only on BIs x_k, $k \neq k^*$, which are fixed by assumption. Therefore, the product term can be factored into $\left(A_{k^*}^\ell \cdot x_{k^*} + B_{k^*}^\ell \right)$ so that constants $A_{k^*}^\ell$ and $B_{k^*}^\ell$ are functions of $\{x_k\}_{\substack{k=1 \\ k \neq k^*}}^n$

$$\max_{\substack{x_k^L \le x_k \le x_k^U\\ k \neq k^*}} \left[\max_{\ell} \left[\max_{x_{k^*} \in S_{k^*}^{\ell}} \sum_{i_1 \in I_1^M} Y_{i_1}^M \cdot \left(\sum \left(A\left(\{x_k\}_{\substack{k=1\\k\neq k^*}}^n \right) \cdot x_{k^*} + B\left(\{x_k\}_{\substack{k=1\\k\neq k^*}}^n \right) \right) \right) \right] \right]$$

As the resultant objective function is linear for x_{k^*}, it is guaranteed that the maximum will occur at either of the endpoints of subinterval $S_{k^*}^{\ell}$ defined for $x_{k^*}^L \le x_{k^*} \le x_{k^*}^U$. A similar procedure to the above can be followed for all other BIs x_k as well as for the minimization problem. Therefore, inspecting the set of points $\{x_k^L, u_k, m_k, t_k, T_k, M_k, U_k, x_k^U\}$ for $\{x_k\}_{k=1}^n$ is sufficient to obtain the solution to Eq. 10.

Algorithm Description

The global optimization algorithm used in this work is a hybrid branch-and-bound/interval analysis scheme similar to the branch-and-bound algorithm and the hybrid branch-and-bound/interval analysis algorithm used by Manousiouthakis and Sourlas.[22,23] Figure 7 illustrates a portion of the algorithm's branching structure where n BIs are defined in $[x_k^L, x_k^U]$, $k \in \{1,2,\ldots,n\}$ such that $[x_1^L, x_1^U]$ contains breakpoints $x_{1,1}$, $x_{1,2}$, $x_{1,3}$ and $[x_2^L, x_2^U]$ contains breakpoints $x_{2,1}$, $x_{2,2}$. We denote $[SI_k^L, SI_k^U]$ as the SII, where $k = 0$ indicates that all BIs $\{x_k\}_{k=1}^n$ are in their interval form, and $k \in \{1,2,\ldots,n\}$ indicates that $\{x_1, x_2,\ldots,x_k\}$ are crisp values (i.e., x_k^L, x_k^U) and $\{x_{k+1}, x_{k+2},\ldots,x_k\}$ are intervals. The following step-by-step algorithm description is for the maximization portion of the optimization problem.

1. Calculate $[\mathrm{SI}_0^L, \mathrm{SI}_0^U]$.
   a. For $k \in \{1,2,\ldots,n\}$, identify edges and breakpoints contained in $[x_k^L, x_k^U]$.
   b. Evaluate $\mu_{k,i_k}^0(x_k)$ at edges and breakpoints to obtain $\left\{ \left[\mu_{k,W}^{0,L}, \mu_{k,W}^{0,U}\right], \left[\mu_{k,M}^{0,L}, \mu_{k,M}^{0,U}\right], \left[\mu_{k,S}^{0,L}, \mu_{k,S}^{0,U}\right] \right\}$, $k \in \{1, 2, \ldots, n\}$.
   c. Determine SI_0^L and SI_0^U from Eqs. 8 and 9 using $\left\{\mu_{k,W}^{0,L}, \mu_{k,M}^{0,L}, \mu_{k,S}^{0,L}\right\}$, $k \in \{1, 2, \ldots, n\}$ and $\left\{\mu_{k,W}^{0,U}, \mu_{k,M}^{0,U}, \mu_{k,S}^{0,U}\right\}$, $k \in \{1, 2, \ldots, n\}$, respectively.
2. Instantiate branch list and store $[\mathrm{SI}_0^L, \mathrm{SI}_0^U]$ and $[x_k^L, x_k^U]$, $k \in \{1,2,\ldots,n\}$ in row 1.
3. Select $[\mathrm{SI}_k^L, \mathrm{SI}_k^U]$ and corresponding intervals/crisp values in row 1.
4. Identify edges and breakpoints contained in $[x_{k+1}^L, x_{k+1}^U]$, the first noncrisp interval.
5. For each edge and breakpoint identified, replace $[x_{k+1}^L, x_{k+1}^U]$ with the edge/breakpoint, calculate $[\mathrm{SI}_{k+1}^L, \mathrm{SI}_{k+1}^U]$ as in step 1, then append $[\mathrm{SI}_{k+1}^L, \mathrm{SI}_{k+1}^U]$ and $[x_k^L, x_k^U]$, $k \in \{1,2,\ldots,n\}$ to the end of the branch list.
6. Rank order all entries on the branch list based on descending order of $[\mathrm{SI}_k^L, \mathrm{SI}_k^U]$ upper bounds.
7. If $k = n$, terminate. The SI_k^U value is the maximum of problem (10). Otherwise, go to step 3.

Finding the solution to the minimization problem quantifying the lower bound of SII involves the same procedure as above, except in step 7, the entries of the branch list are rank ordered by ascending order of $[\mathrm{SI}_k^L, \mathrm{SI}_k^U]$ lower bounds.

Case Study: Preliminaries

Consider a hypothetical company A. The BIs of corporate ecological sustainability to be used for testing our global optimization approach will be the same set used by Phillis and Davis.[14] Each of these is categorized under one of six indicators: AIR, LAND, WATER, POLICY, HEALTH, and WEALTH. AIR, LAND, and WATER comprise the indicator ECOS (ecosystem sustainability), and POLICY, HEALTH, and WEALTH comprise the indicator HUMS (human sustainability). The six BIs for company A, with units in parentheses, are listed in Table 1, and a tree diagram showing how the indicators for ECOS are combined is illustrated in Figure 8.

Three sets of intervals of the BIs, given in Table 2, were chosen to test the algorithm used to solve our global

Table 1. Corporate Ecosystem Sustainability Indicators

AIR	GHG (x_1)	Greenhouse gas emissions (kg CO_2/million dollars of annual net sales)
	TX (x_2)	EPA toxic releases (g/L)
LAND	SW (x_3)	Solid waste generation (g/L)
	RECY (x_4)	Solid waste recycled (%)
	HW (x_5)	Hazardous waste generated (g/L)
WATER	WATER (x_6)	Water use per unit of production (L water/L produced)
POLICY	PENB (x_7)	Pension benefits paid to employees ($)
	ECAU (x_8)	Percentage of employees covered by bargaining agreements and unions (%)
	CCOM (x_9)	Financial and other contributions to communities (% annual sales)
	TAX (x_{10})	Taxes paid to government (%)
WEALTH	RPE (x_{11})	Sales revenue per employee (million $ per year)
	DEBT (x_{12})	Debt ratio (Total debt/Total assets)
HEALTH	LTIR (x_{13})	Lost time injury rate (# incidences per 100 employees per year)
	IR (x_{14})	Injury rate (# incidences per 100 employees per year)
	LWD (x_{15})	Lost workdays (# days per employee per year)
	HLIB (x_{16})	Health and life insurance benefits (thousand $ per employee per year)

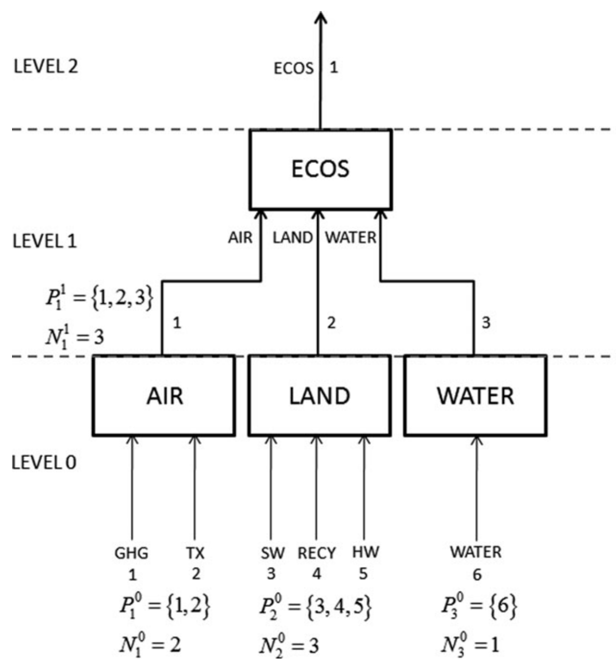

Figure 8. Tree diagram for Example 1.

Table 2. BI Intervals for Case Study

	Case 1 $[x_k^L, x_k^U]$	Case 2 $[x_k^L, x_k^U]$	Case 3 $[x_k^L, x_k^U]$
GHG (x_1)	[300, 700]	[330, 335]	[154, 231.3]
TX (x_2)	[3, 9]	[13.8, 13.9]	[0.1328, 0.1992]
SW (x_3)	[2000, 2500]	[1670, 1680]	[0.1888, 0.2832]
RECY (x_4)	[0.4, 0.8]	[0.79, 0.8]	[0.776, 1.164]
HW (x_5)	[0.01, 0.06]	[0.027, 0.028]	[0.01, 0.015]
WATER (x_6)	[5.2, 5.8]	[5.38, 5.42]	[4.5936, 6.8904]
PENB (x_7)	[11,000, 12,000]	[6500, 6600]	[5443, 8165]
ECAU (x_8)	[0.3, 0.4]	[0.34, 0.35]	[0.4112, 0.6168]
CCOM (x_9)	[0.001, 0.003]	[0.0019, 0.002]	[0.00094, 0.00141]
TAX (x_{10})	[0.3, 0.5]	[0.2, 0.22]	[0.1474, 0.221]
RPE (x_{11})	[0.5, 0.6]	[0.41, 0.42]	[0.493, 0.739]
DEBT (x_{12})	[0.15, 0.6]	[0.188, 0.189]	[0.426, 0.64]
LTIR (x_{13})	[0.1, 0.2]	[0.267, 0.268]	[0.36, 0.54]
IR (x_{14})	[1, 2]	[1.6, 1.75]	[3.05, 4.57]
LWD (x_{15})	[0.1, 0.3]	[0.108, 0.11]	[0.252, 0.378]
HLIB (x_{16})	[0.5, 0.7]	[1.0, 1.01]	[1.859, 2.789]

optimization problem. The first set was chosen such that the intervals contain different sets of breakpoints. The second set involves tight intervals encapsulating only the M_k and m_k breakpoints. The last set is derived from data contained in Ref. 14, with a ±20% range applied to simulate the uncertainty in actual corporate data. For this study, $\alpha_k = 0.6$, $k \in \{1,2,\ldots,16\}$. The values of u_k, m_k, t_k, T_k, M_k, U_k, $k \in \{1,2,\ldots,16\}$ along with the type of normalization function used for each of the six BIs are given in Table 3.

To clarify the procedure for obtaining the level 0 membership grade intervals, Figure 9 illustrates how the TAX BI interval $[x_{10}^L, x_{10}^U] = [0.3, 0.5]$ for Case 1 translates into membership grade intervals $\left[\mu_{10,i_k}^{0L}, \mu_{10,i_k}^{0U}\right]$, $i_k = \{M, S\}$. The membership grade interval $[\mu_{10,W}^{0L}, \mu_{10,W}^{0U}]$ is not shown because it is clear in Figure 9 that $\mu_{10,W}^{0}(x_{10}) = 0 \in \mathrm{x}_{10} \in [0.3,0.5]$. Table 4 shows the membership grade intervals for the level 0 linguistic variables for the BI intervals of Case 1, Examples 1 and 2.

Once the level 0 membership grades are determined, rule sets R_{j,i_j}^0, $j \in \{1, 2, ..., 6\}$ determine the corresponding level 1 membership grades to linguistic variables VB = "very bad," B = "bad," A = "average," G = "good," and VG = "very good." Using these intermediate membership grades, rule sets R_{1,i_1}^1, R_{2,i_2}^1 (for ECOS and HUMS, respectively) determine the corresponding level 2 membership grades to linguistic variables EL = "extremely low," VL = "very low," L = "low," FL = "fairly low," I = "intermediate," FH = "fairly high," H = "high," VH = "very high," and EH = "extremely high." The aforementioned rule sets, sorted by number of inputs, are given in the Appendix. The peak values of the level 2 membership grade functions, illustrated in Figure 10, are used in the final SII calculation for ECOS and HUMS.

Case Study: Computational Results

The ECOS and HUMS SIIs for company A calculated by the branch-and-bound algorithm are given in Table 5 below and are illustrated in Figure 11. All computations were done on a quad-core 2.4 GHz PC using a MATLAB implementation of the proposed algorithm. Example 1 of the case study uses 46 variables ($6x_k'$s, 18 $\mu_{k,i_k}^0 s$, 13 $\mu_{k,i_k}^1 s$, 9 $\mu_{1,i_1}^2 s$) with 40 equality constraints. Example 2 uses 64 variables (10 $x_k' s$, 30 $\mu_{k,i_k}^0 s$, 15 $\mu_{k,i_k}^1 s$, 9 $\mu_{1,i_1}^2 s$) with 54 equality constraints. Thus, there are 6 and 10 degrees of freedom in Examples 1 and 2, respectively, corresponding to the number of BIs.

The regions containing all possible optimal vectors for the minima and maxima of the ECOS and HUMS intervals for all cases are given in Tables 6 and 7, respectively.

M_1, M_2, M_3, m_4, M_5, and M_6 do not appear in any of the optimal regions for Example 1. The above suggests that if at least one BI interval is defined such that $\left(x_k^L < u_k\right) \vee \left(x_k^U < u_k\right)$ or $\left(x_k^L < U_k\right) \vee \left(x_k^U < U_k\right)$ for the minimum and $x_k^L \in [t_k, T_k] \vee x_k^U \in [t_k, T_k]$ for the maximum, there exist an infinite number of minimum and maximum solutions for the considered optimization problems. Nevertheless, the proposed branch-and-bound/interval analysis algorithm is able to efficiently quantify the intervals in which the coordinates of the optimal solutions belong. In terms of decision making, this property of the optimal vectors has significant implications: if the optimal vectors contain two values for a certain BI, then improvement in that BI (either by increasing the minimum or reducing the maximum of the BI in accordance to its goal) is not guaranteed to result in an improvement to company A's ECOS SII. For Case 1, this holds for GHG, RECY, and HW.

Case 2 of both examples results in a finite number of solutions and implies that improvement in any of the BIs for this case will result in an improvement to company A's ECOS SII. Even though the BIs were defined as tight intervals containing M_k and m_k terms, the optimal vectors do not contain any M_k's or m_k's. The resulting ECOS SII of Example 1 is a tight interval containing the value 0.5, the midpoint of the

Table 3. Normalization Parameters for BIs

Indicator	Function	u_k	m_k	t_k	T_k	M_k	U_k
GHG	Figure 2a	N/A	N/A	N/A	100	332.8	682
TX	Figure 2a	N/A	N/A	N/A	6.54	13.864	24.85
SW	Figure 2a	N/A	N/A	N/A	787	1677.4	3013
RECY	Figure 2b	0.5	0.794	0.99	N/A	N/A	N/A
HW	Figure 2a	N/A	N/A	N/A	0.0147	0.028	0.0479
WATER	Figure 2a	N/A	N/A	N/A	5	5.4	6
PENB	Figure 2c	1000	6526	10,210	13,690	18,321	25,268
ECAU	Figure 2b	0	0.348	0.58	N/A	N/A	N/A
CCOM	Figure 2b	0.001	0.00196	0.0026	N/A	N/A	N/A
TAX	Figure 2c	0	0.21	0.35	0.35	0.61	1
RPE	Figure 2b	0.212	0.413	0.547	N/A	N/A	N/A
DEBT	Figure 2c	0	0.1884	0.314	0.314	0.5884	1
LTIR	Figure 2a	N/A	N/A	N/A	0	0.2676	0.669
IR	Figure 2a	N/A	N/A	N/A	0	1.696	4.24
LWD	Figure 2a	N/A	N/A	N/A	0	0.1092	0.273
HLIB	Figure 2b	0	1.006	1.677	N/A	N/A	N/A

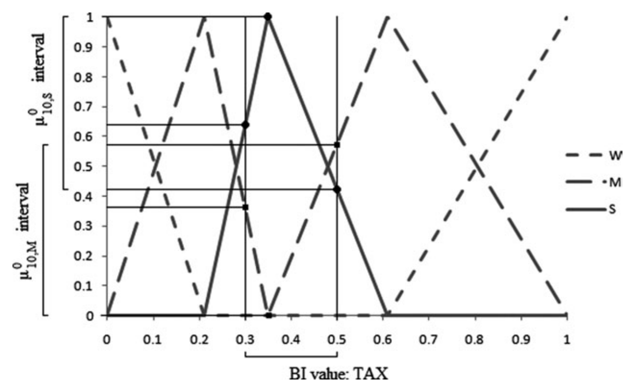

Figure 9. BI to membership grade conversion.

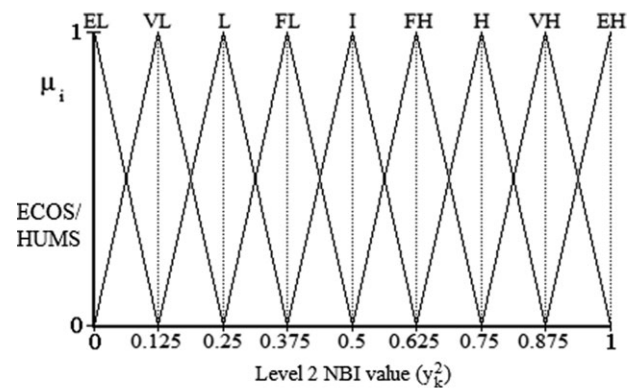

Figure 10. Level 2 membership grade functions.

domain of SII values. This result is consistent with the definition of the level 0 fuzzy set "medium." However, in Example 2, the HUMS SII is tight and contains the value 0.25, reflecting a more pessimistic system of rule sets.

As $x_2^U < T_2$ and $x_3^U < T_3$ for Case 3, indicating that TX and SW already have the highest sustainability rating, the optimal region contains all values in the corresponding BI intervals. It is clear that improvement in TX and SW will not change company A's ECOS SII in any way, while an improvement in WATER may garner no change in the ECOS SII. Although the GHG interval does not include the highest possible value, the maximum ECOS SII value is still 1. Examination of the rule set in Table A5 reveals why this anomaly occurs: rule 122 assigns the level 2 linguistic variable i_1 = "EH" to the level 1 linguistic variable combination (i_1 = "G", i_2 = "VG", i_3 = "VG"), implying that the AIR interval needs not contain the maximum value of 1 to yield an ECOS interval that contains 1.

Case 1 of Example 2 produces the most interesting outcomes in its region of optimal vectors: the optimal region includes m_8 and the strict interior points $t_{10} = T_{10} = 0.35 \in [0.3, 0.5]$ and $t_{12} = T_{12} = 0.314 \in [0.15, 0.6]$. The latter point implies that the SII objective function is not monotonic with respect to BIs x_{10}, x_{12}. The former is easily explained by inspection of the $\mu^0_{k,S}$ intervals in Table 4 and the rule sets in Table A4: to maximize SII, $\mu^0_{7,S} = \mu^0_{9,S} = \mu^0_{10,S} = 1$, and since any combination of three "S" grades and either "M" or "S" maps to the level 1 linguistic variable "VG" (by rules 77–81), it is possible for $\mu^0_{8,M} = 1$ at the maximum.

Table 4. Membership Grade Intervals for Case 1, Examples 1 and 2

BI	$\mu^0_{k,W}$ Interval	$\mu^0_{k,M}$ Interval	$\mu^0_{k,S}$ Interval
GHG	[0,1]	[0,1]	[0,0.1409]
TX	[0,0]	[0,0.3359]	[0.6641,1]
SW	[0.2415,0.6159]	[0.3841,0.7585]	[0,0]
RECY	[0,1]	[0,1]	[0,0.0306]
HW	[0,1]	[0,1]	[0,1]
WATER	[0,0.6667]	[0.3333,1]	[0,0.5]
PENB	[0,0]	[0,0]	[1,1]
ECAU	[0,0.1379]	[0.7759,1]	[0,0.2241]
CCOM	[0,1]	[0,1]	[0,1]
TAX	[0,0]	[0,0.5769]	[0.4231,1]
RPE	[0,0]	[0,0.3507]	[0.6493,1]
DEBT	[0,0.2038]	[0,1]	[0,1]
LTIR	[0,0]	[0.3737,0.7474]	[0.2526,0.6263]
IR	[0,0.1195]	[0.5896,1]	[0,0.4104]
LWD	[0,1]	[0,1]	[0,0.0842]
HLIB	[0.3043, 0.5031]	[0.4969, 0.6957]	[0,0]

Case Study: Finite Variation Analysis

Although the preceding SII calculations are effective in identifying where company A stands in its sustainability efforts, they alone are not sufficient in determining which BI should the company A try to change so as to improve its overall ecological and human sustainability rating. Consider Example 1 Case 3: a simple analysis of the BI intervals would suggest that company A should improve its water usage, as the upper bound of the BI interval for WATER (x_6) exceeds U_6. What this analysis fails to consider is how much change in each of the BI intervals is necessary to produce a tangible improvement in company A's ECOS SII. Although company A's Example 1 Case 3 WATER BI is clearly the worst of the BIs, its improvement may require considerably more effort in optimizing water usage compared to other areas. This statement holds for any BI whose optimal regions contain an infinity of solutions.

To address this issue in the decision-making process regarding company A's sustainability efforts, a finite variation analysis is pursued for the three cases considered in this article. This analysis is carried out by first defining a "gap" as the difference of the BI maximum and minimum. Then, a "10% gap improvement" is defined as follows: if a lower value is desired for some BI interval, then the maximum is reduced by 10% of the gap; if a higher value is desired, then the minimum is increased by 10% of the gap; if a middle value is desired, then either the minimum and maximum is changed by 10% of the gap depending on which changes the minimum SII value the most. Consider that a 10% gap improvement is done for each BI interval, while the other BI intervals are kept fixed. Then, the new ECOS SII and HUMS SII are listed in Tables 8 and 9 below for Examples

Table 5. Results of Branch-and-Bound/Interval Analysis Algorithm

	Case 1		Case 2		Case 3	
	Min	Max	Min	Max	Min	Max
Example 1						
SII value	0.1813	0.7509	0.4885	0.5223	0.5543	1
Run time (s)	0.615	0.764	0.382	0.39	0.202	0.211
Iterations	470	586	359	365	144	144
Branches	1144	1376	717	729	480	480
Worst case	1440 branches		729 branches		480 branches	
Example 2						
SII value	0.3182	0.8579	0.2500	0.2869	0.2455	0.5422
Run time (s)	326	316	77.9	19.8	11.1	7.21
Iterations	10,762	11,300	6234	3247	2794	2326
Branches	26,640	27,178	12,467	6493	4854	3920
Worst case	51,840 branches		59,049 branches		26,244 branches	

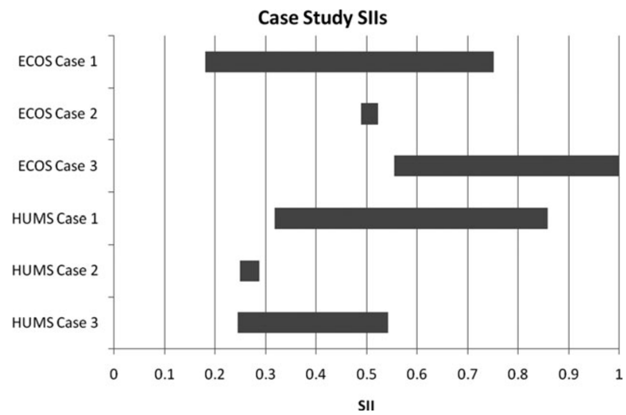

Figure 11. SIIs for company A.

1 and 2, respectively. "BASE CASE" refers to the original set of BI intervals as shown in Table 2.

For all cases in both examples, only the lower bounds of the calculated ECOS SII and HUMS SII have changed, whereas the upper bounds remained unaltered. This is not unreasonable for the gap improvement considered.

As predicted, improvement in RECY and HW did not improve the ECOS SII in Case 1; however, the ECOS SII corresponding to a 10% gap improvement to GHG still changed despite the minimum vectors having two values for that BI. This is because the 10% gap improvement reduced the maximum of GHG to 660, which is less than $U_1 = 682$ and thus the GHG NBI no longer contains 0. The recommended course of action for company A to improve its sustainability is to focus first on its water usage, because its improvement results in the largest increase in the ECOS SII lower bound, and then focus on its EPA toxic releases and greenhouse gas emissions. Case 2's results reaffirm the conclusion reached via examination of its optimal vectors. As company A's ECOS SII is intermediate, and improvement in all six BI intervals results in an improvement in the ECOS SII, the recommended course of action would be to look into all aspects of its manufacturing process to pinpoint areas of improvement, starting with water usage and hazardous waste generation. In Case 3, TX, SW, and WATER did not improve after a 10% gap improvement was applied. Given these results, company A should start its efforts on improving its ecological sustainability by looking into its recycling operations first, then greenhouse gas emissions and hazardous waste generation.

The results for Example 2 indicate that company A should focus first on increasing its contributions to the community in Case 1 and increase its sales revenue per employee in Case 3. However, no tangible increase in the minimal SII value is achieved with a 10% gap improvement to any BI in Case 2. Therefore, in this case, company A must expend considerably more effort to bring about an improvement in its SII.

Table 6. Regions of Minimum/Maximum Vectors for Example 1

Case 1	Min	$[U_1, x_1^U], x_2^U, x_3^U, [x_4^L, u_4], [U_5, x_5^U], x_6^U$
	Max	$x_1^L, [x_2^L, T_2], x_3^L, x_4^U, [x_5^L, T_5], x_6^L$
Case 2	Min	$x_1^U, x_2^U, x_3^U, x_4^L, x_5^U, x_6^U$
	Max	$x_1^L, x_2^L, x_3^L, x_4^U, x_5^L, x_6^L$
Case 3	Min	$x_1^U, [x_2^L, x_2^U], [x_3^L, x_3^U], x_4^L, x_5^U, [U_6, x_6^U]$
	Max	$[x_1^L, x_1^U], [x_2^L, x_2^U], [x_3^L, x_3^U], [t_4, x_4^U], [x_5^L, T_5], [x_6^L, T_6]$

Intermediate SII Calculations

To demonstrate the reducibility of the SII optimization problem, the intermediate AIR SII is determined. Figure 12 illustrates the level 1 membership grade functions for AIR, whose peak values are used in the objective function for calculation of lower and upper bounds for the AIR SII.

The reduced global optimization problem is given in problem (11) below.

$$\min, \max\ 0\cdot\mu_{1,\mathrm{VB}}^1 + 0.25\cdot\mu_{1,\mathrm{B}}^1 + 0.5\cdot\mu_{1,\mathrm{A}}^1 + 0.75\cdot\mu_{1,\mathrm{G}}^1 + 1\cdot\mu_{1,\mathrm{VG}}^1$$

$s.t.$

Level 0 to level 1 rule sets for AIR :

$$\mu_{1,\mathrm{VB}}^1 = \mu_{1,\mathrm{W}}^0 \cdot \mu_{2,\mathrm{W}}^0$$
$$\mu_{1,\mathrm{B}}^1 = \mu_{1,\mathrm{W}}^0 \cdot \mu_{2,\mathrm{M}}^0 + \mu_{1,\mathrm{M}}^0 \cdot \mu_{2,\mathrm{W}}^0$$
$$\mu_{1,\mathrm{A}}^1 = \mu_{1,\mathrm{W}}^0 \cdot \mu_{2,\mathrm{S}}^0 + \mu_{1,\mathrm{M}}^0 \cdot \mu_{2,\mathrm{M}}^0 + \mu_{1,\mathrm{S}}^0 \cdot \mu_{2,\mathrm{W}}^0$$
$$\mu_{1,\mathrm{G}}^1 = \mu_{1,\mathrm{M}}^0 \cdot \mu_{2,\mathrm{S}}^0 + \mu_{1,\mathrm{S}}^0 \cdot \mu_{2,\mathrm{M}}^0$$
$$\mu_{1,\mathrm{VG}}^1 = \mu_{1,\mathrm{S}}^0 \cdot \mu_{2,\mathrm{S}}^0$$

Level 0 membership grades :

$$\mu_{k,i_k}^0 = \mu_{k,i_k}^0(x_k),\quad k \in \{1,2\},\ i_k \in \{W, M, S\}$$

Membership function constraints :

$$\mu_{1,\mathrm{VB}}^1 + \mu_{1,\mathrm{B}}^1 + \mu_{1,\mathrm{A}}^1 + \mu_{1,\mathrm{G}}^1 + \mu_{1,\mathrm{VG}}^1 = 1$$
$$\mu_{k,W}^0 + \mu_{k,W}^0 + \mu_{k,W}^0 = 1,\ k \in \{1,2\}$$

Inequality constraints :

$$x_k^L \le x_k \le x_k^U,\quad k \in \{1,2\} \tag{11}$$

The proposed branch-and-bound/interval analysis algorithm can easily be applied to this reduced problem, and it yields an AIR SII of [0.416, 0.7852], [0.4976, 0.5052], and [0.859, 0.9418] for cases 1, 2, and 3, respectively, of Example 1.

Discussion/Conclusions

In case 3, the final ECOS SII had a maximum of 1, even though the AIR interval did not contain the highest possible value of 1. If such a possibility is considered as an undesirable outcome of the sustainability assessment process, then one can define a level m rule set such that the highest possible level $m + 1$ linguistic variable is only given to the combination in which all level m linguistic variables have the highest possible values. Also, the results of the finite variation analysis for case 3 confirm the suspicions raised regarding "hasty" decision making based solely on examining the BI intervals. Even though company A's WATER BI interval is clearly the

Table 7. Regions of Minimum/Maximum Vectors for Example 2

Case 1	Min	$[x_7^L, x_7^U], x_8^L, x_9^L, x_{10}^U, x_{11}^L, x_{12}^L, x_{13}^U, x_{14}^U, [U_{15}, x_{15}^U], x_{16}^L$
	Max	$[x_7^L, x_7^U], [m_8, x_8^U], [t_9, x_9^U], \{t_{10} = T_{10}\}, x_{11}^U,$ $\{t_{12} = T_{12}\}, x_{13}^L, x_{14}^L, x_{15}^L, x_{16}^U$
Case 2	Min	$x_7^L, x_8^L, x_9^L, x_{10}^L, x_{11}^L, x_{12}^L, x_{13}^U, x_{14}^U, x_{15}^U, x_{16}^L$
	Max	$x_7^U, x_8^U, x_9^U, x_{10}^U, x_{11}^U, x_{12}^U, x_{13}^L, x_{14}^L, x_{15}^L, x_{16}^U$
Case 3	Min	$x_7^L, x_8^L, x_9^L, x_{10}^L, x_{11}^L, x_{12}^U, x_{13}^U, [U_{14}, x_{14}^U],$ $[U_{15}, x_{15}^U], [x_{16}^L, x_{16}^U]$
	Max	$x_7^U, [t_8, x_8^U], x_9^U, x_{10}^U, x_{11}^U, x_{12}^L, x_{13}^L, x_{14}^L, x_{15}^L, [x_{16}^L, x_{16}^U]$

Table 8. Finite Variation Analysis: Example 1

	Case 1		Case 2		Case 3	
	BI interval	ECOS SII	BI interval	ECOS SII	BI interval	ECOS SII
BASE CASE	N/A	[0.1813, 0.7509]	N/A	[0.4885, 0.5223]	N/A	[0.5543, 1]
Improve GHG	[300,660]	[0.1873, 0.7509]	[330,334.5]	[0.4887, 0.5223]	[154.2, 223.59]	[0.5585, 1]
Improve TX	[3,8.4]	[0.1895, 0.7509]	[13.8,13.89]	[0.4886, 0.5223]	[0.1328,0.19256]	[0.5543, 1]
Improve SW	[2000,2450]	[0.1855, 0.7509]	[1670,1679]	[0.4886, 0.5223]	[0.1888,0.27376]	[0.5543, 1]
Improve RECY	[0.44,08]	[0.1813, 0.7509]	[0.791, 0.8]	[0.4889, 0.5223]	[0.8148,1.164]	[0.5618, 1]
Improve HW	[0.01,0.055]	[0.1813, 0.7509]	[0.027, 0.0279]	[0.4894, 0.5223]	[0.01,0.0145]	[0.5545, 1]
Improve WATER	[5.2,5.74]	[0.1964, 0.7509]	[5.38,5.416]	[0.4902, 0.5223]	[4.5936,6.66072]	[0.5543, 1]

Table 9. Finite Variation Analysis: Example 2

	Case 1		Case 2		Case 3	
	BI Interval	HUMS SII	BI Interval	HUMS SII	BI Interval	HUMS SII
BASE CASE	N/A	[0.3182, 0.8579]	N/A	[0.2500, 0.2869]	N/A	[0.2455, 0.5422]
Improve PENB	[11100, 12000]	[0.3182, 0.8579]	[6510, 6600]	[0.2500, 0.2869]	[5715.1, 8165]	[0.2474, 0.5422]
Improve ECAU	[0.31, 0.4]	[0.3193, 0.8579]	[0.341, 0.35]	[0.2500, 0.2869]	[0.43176, 0.6168]	[0.2477, 0.5422]
Improve CCOM	[0.0012, 0.003]	[0.3387, 0.8579]	[0.00191, 0.002]	[0.2500, 0.2869]	[0.000987, 0.00141]	[0.2455, 0.5422]
Improve TAX	[0.3, 0.48]	[0.3246, 0.8579]	[0.202, 0.220]	[0.2500, 0.2869]	[0.15476, 0.221]	[0.2470, 0.5422]
Improve RPE	[0.51, 0.6]	[0.3244, 0.8579]	[0.411, 0.42]	[0.2500, 0.2869]	[0.5176, 0.739]	[0.2466, 0.5422]
Improve DEBT	[0.195, 0.6]	[0.3246, 0.8579]	[0.1881, 0.189]	[0.2500, 0.2869]	[0.426, 0.6186]	[0.2493, 0.5422]
Improve LTIR	[0.1, 0.19]	[0.3198, 0.8579]	[0.267, 0.2679]	[0.2500, 0.2869]	[0.36, 0.522]	[0.2486, 0.5422]
Improve IR	[1, 1.9]	[0.3199, 0.8579]	[1.6, 1.735]	[0.2500, 0.2869]	[3.05, 4.418]	[0.2455, 0.5422]
Improve LWD	[0.1, 0.28]	[0.3182, 0.8579]	[0.108, 0.1098]	[0.2500, 0.2869]	[0.252, 0.3654]	[0.2455, 0.5422]
Improve HLIB	[0.52, 0.7]	[0.3194, 0.8579]	[1.001, 1.01]	[0.2500, 0.2869]	[1.952, 2.789]	[0.2455, 0.5422]

culprit in this case, a 10% gap improvement in that interval is not sufficient to improve its ECOS SII, while 10% gap improvements in recycling, hazardous waste generation, and greenhouse gas emissions each produced tangible benefits to the ECOS SII. Therefore, a reasonable decision-making sustainability analysis should calculate not only where an entity stands in its sustainability efforts but also how its sustainability rating will change with certain improvements.

In the preceding case study, we have seen how a set of BI intervals is transformed into the final SII via a combination of trapezoidal normalization functions, triangular membership grade functions, and rule sets assigning a level m linguistic variable to all rule-related possible combinations of level $m-1$ linguistic variables. The computation of SII is shown to be possible through solution of a minimization and a maximization problem. Both of these problems are nonconvex and require global optimization techniques for their solution. A number of properties for these problems are theoretically established. These properties are capitalized upon in developing a branch-and-bound algorithm that can identify the global optima for both the minimization and maximization problems considered. Through proof and application, we have confirmed that the minima and maxima of the global optimization problem occur either at the breakpoints of the normalization and membership grade functions or at the BI interval edges. Because of the pure mathematical nature of our conceptual framework, if someone were given the functions, rule sets, and BI intervals used in our case study, he or she would be able to exactly reproduce our results using our hybrid branch-and-bound/interval analysis algorithm. We have also demonstrated how calculation of the SII and subsequent finite variation analysis on the SII bounds facilitates an entity's decision making with regard to its sustainability.

The conceptual framework of SII detailed in this work brings forth numerous avenues of research. Acceleration of the proposed algorithm may be possible in light of some of the theoretical developments in Ref. 24. At the theoretical level, cases where the BI values must lie within a polytope instead of a hyper-rectangle can be explored. Also, if probabilistic information better characterizes the uncertainty of some BIs, the proposed analysis can still be carried out in the following context. The "probabilistic" BIs can be considered to belong to $1-\sigma$, $2-\sigma$, $3-\sigma$ standard deviation intervals. Then, the interval analysis is repeatedly carried out and $1-\sigma$, $2-\sigma$, $3-\sigma$ SIIs are calculated. Furthermore, if time series data are available for the BIs, then they can be translated to a time-dependent SII. The time evolution of SII will therefore allow conclusions to be reached regarding the effectiveness, or lack thereof, of actions taken by the entity toward increased sustainability. Finally, the proposed SII framework can be used within an embedded optimization problem formulation to pursue the globally optimal synthesis of sustainable systems. One example is the identification of minimum-cost "sustainability improvement plans" for a set of related entities via embedded global optimization, in which a cost function related to design changes is minimized, subject to

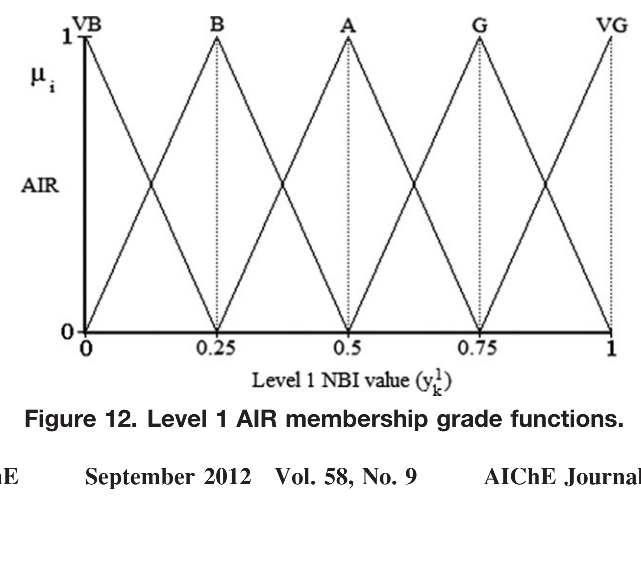

Figure 12. Level 1 AIR membership grade functions.

improvement-related constraints on the SII upper and lower bounds and BI interval bounds. This problem formulation has the potential of differentiating among two or more entities even in cases where their SIIs overlap, as it focuses exclusively on changing the interval edges of the SIIs.

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Notation

Letters

A = level 1 fuzzy set corresponding to linguistic variable "average"
B = level 1 fuzzy set corresponding to linguistic variable "bad"
EH = level 2 fuzzy set corresponding to linguistic variable "extremely high"
EL = level 2 fuzzy set corresponding to linguistic variable "extremely low"
FH = level 2 fuzzy set corresponding to linguistic variable "fairly high"
EL = level 2 fuzzy set corresponding to linguistic variable "fairly low"
G = level 1 fuzzy set corresponding to linguistic variable "good"
H = level 2 fuzzy set corresponding to linguistic variable "high"
I = level 2 fuzzy set corresponding to linguistic variable "intermediate"
I_k^m = the collection of fuzzy sets used for level m indicator k
L = level 2 fuzzy set corresponding to linguistic variable "low"
m_k = the basic indicator value in $[u_k, t_k]$, $1 \leq k \leq n$ for which $y_k = \alpha_k$
M = level 0 fuzzy set corresponding to linguistic variable "medium"
M_k = the basic indicator value in $[T_k, U_k]$, $1 \leq k \leq n$ for which $y_k = \alpha_k$
N^m = the cardinality (number of members) of set P^m
N_j^m = the cardinality of set P_j^m
P^m = the index set of level m inference engines mapping m to $m + 1$
P_j^m = the index set of inputs to level m inference engine j
$R_{j,ij}^m$ = the rule set of all level m fuzzy set n-tuples $(i_1,i_2,\ldots,i_n)$ of inference engine j that map to the fuzzy set i_j of level $m + 1$
S = level 0 fuzzy set corresponding to linguistic variable "strong"
t_k = minimum target value of basic indicator k
T_k = maximum target value of basic indicator k
u_k = minimum value of basic indicator k for which y_k is nonzero
U_k = maximum value of basic indicator k for which y_k is nonzero
VB = level 1 fuzzy set corresponding to linguistic variable "very bad"
VG = level 1 fuzzy set corresponding to linguistic variable "very good"
VH = level 2 fuzzy set corresponding to linguistic variable "very high"
VL = level 2 fuzzy set corresponding to linguistic variable "very low"
W = level 0 fuzzy set corresponding to linguistic variable "weak"
x_k = value of basic indicator k in $[x_k^L, x_k^U]$, $1 \leq k \leq n$
x_k^L = minimum value of basic indicator k
x_k^U = maximum value of basic indicator k
y_k^m = level m normalized value of basic indicator k in $[y_k^L, y_k^U]$, $1 \leq k \leq n$
y_k^{mL} = level m minimum normalized value of basic indicator k
y_k^{mU} = level m maximum normalized value of basic indicator k
Y_{ik}^m = peak value for the level m fuzzy set i_k, i.e., the value at which the membership grade value assumes its maximum

Greek letters

α_k = the kth normalized basic indicator value for which $\mu_{k,M}^0$ is maximized
$\mu_{k,ik}^m$ = level m membership grade to fuzzy set $i_k \in I_k^m$
$\mu_{j,ij}^{m+1}$ = level $m + 1$ membership grade to fuzzy set $i_j \in I_j^{m+1}$ from inference engine j, $m \in \{0,1,\ldots,M - 1\}$

Subscripts

i_j = fuzzy set of output from inference engine j
i_k = fuzzy set of indicator k
i_p = n-tuple of fuzzy sets feeding inference engine j, $p \in P_j^m$
j = inference engine index
k = indicator index
n = cardinality of set P_j^m, $n = P_j^m$

Superscripts

L = lower bound
m = level of sustainability analysis
U = upper bound

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Appendix: Rule Sets Used in Case Study

Tables A1–A4 show the level 0 rule sets for inference engines using one, two, three, and four inputs, respectively. Table A5 shows the level 1 rule set used to calculate the ECOS and HUMS SII.

Table A1. Level 0 Rule Set for Inference Engines with One BI Input (WATER)

Rule	6 (WATER)	3 (WATER)
1	W	VB
2	M	A
3	S	VG

Table A2. Level 0 Rule Set for Inference Engines with Two BI Inputs (AIR and WEALTH)

Rule	1 (GHG)/ 11 (RPE)	2 (TX)/ 12 (DEBT)	1 (AIR)/ 5 (WEALTH)
1	W	W	VB
2	W	M	B
3	M	W	B
4	W	S	A
5	M	M	A
6	S	W	A
7	M	S	G
8	S	M	G
9	S	S	VG

Table A3. Level 0 Rule Set for Inference Engines with Three BI Inputs (LAND)

Rule	3 (SW)	4 (RECY)	5 (HW)	2 (LAND)
1	W	W	W	VB
2	W	W	M	B
3	W	M	W	B
4	M	W	W	B
5	W	M	M	B
6	M	W	M	B
7	M	M	W	B
8	W	W	S	B
9	W	S	W	B
10	S	W	W	B
11	W	M	S	A
12	W	S	M	A
13	M	W	S	A
14	S	W	M	A
15	M	S	W	A
16	S	M	W	A
17	M	M	M	A
18	W	S	S	G
19	S	W	S	G
20	S	S	W	G
21	M	M	S	G
22	M	S	M	G
23	S	M	M	G
24	M	S	S	G
25	S	M	S	G
26	S	S	M	G
27	S	S	S	VG

Table A4. Level 0 Rule Set for Inference Engines with Four BI Inputs (POLICY and HEALTH)

Rule	7 (PENB)/ 13 (LTIR)	8 (ECAU)/ 14 (IR)	9 (CCOM)/ 15 (LWD)	10 (TAX)/ 16 (HLIB)	4 (POLICY)/ 6 (HEALTH)
1	W	W	W	W	VB
2	W	W	W	M	VB
3	W	W	W	S	VB
4	W	W	M	W	VB
5	W	W	M	M	VB
6	W	W	S	W	VB
7	W	M	W	W	VB
8	W	M	W	M	VB
9	W	M	M	W	VB
10	W	S	W	W	VB
11	M	W	W	W	VB
12	M	W	W	M	VB
13	M	W	M	W	VB
14	M	M	W	W	VB
15	S	W	W	W	VB
16	W	W	M	S	B
17	W	W	S	M	B
18	W	W	S	S	B
19	W	M	W	S	B
20	W	M	M	M	B
21	W	M	M	S	B
22	W	M	S	W	B
23	W	M	S	M	B
24	W	S	W	M	B
25	W	S	W	S	B
26	W	S	M	W	B
27	W	S	M	M	B
28	W	S	S	W	B
29	M	W	W	S	B
30	M	W	M	M	B
31	M	W	M	S	B
32	M	W	S	W	B
33	M	W	S	M	B
34	M	M	W	M	B
35	M	M	W	S	B
36	M	M	M	W	B
37	M	M	M	M	B
38	M	M	S	W	B
39	M	S	W	W	B
40	M	S	W	M	B
41	M	S	M	W	B
42	S	W	W	M	B
43	S	W	W	S	B
44	S	W	M	W	B
45	S	W	M	M	B
46	S	W	S	W	B
47	S	M	W	W	B
48	S	M	W	M	B
49	S	M	M	W	B
50	S	S	W	W	B
51	W	M	S	S	A
52	W	S	M	S	A
53	W	S	S	M	A
54	M	W	S	S	A
55	M	M	M	S	A
56	M	M	S	M	A
57	M	S	W	S	A
58	M	S	M	M	A
59	M	S	S	W	A
60	S	W	M	S	A
61	S	W	S	M	A
62	S	M	W	S	A
63	S	M	M	M	A
64	S	M	S	W	A
65	S	S	W	M	A
66	S	S	M	W	A
67	W	S	S	S	G
68	M	M	S	S	G
69	M	S	M	S	G
70	M	S	S	M	G
71	S	W	S	S	G
72	S	M	M	S	G
73	S	M	S	M	G
74	S	S	W	S	G
75	S	S	M	M	G
76	S	S	S	W	G
77	M	S	S	S	VG
78	S	M	S	S	VG
79	S	S	M	S	VG
80	S	S	S	M	VG
81	S	S	S	S	VG

Table A5. Level 1 Rule Set for ECOS and HUMS

Rule	1 (AIR)/ 4 (POLICY)	2 (LAND)/ 5 (WEALTH)	3 (WATER)/ 6 (HEALTH)	1 (ECOS)/ 2 (HUMS)	Rule	1 (AIR)/ 4 (POLICY)	2 (LAND)/ 5 (WEALTH)	3 (WATER)/ 6 (HEALTH)	1 (ECOS)/ 2 (HUMS)
1	VB	VB	VB	EL	64	A	G	B	I
2	VB	VB	B	EL	65	A	VG	VB	I
3	VB	B	VB	EL	66	G	VB	G	I
4	B	VB	VB	EL	67	G	B	A	I
5	VB	VB	A	VL	68	G	A	B	I
6	VB	B	B	VL	69	G	G	VB	I
7	VB	A	VB	VL	70	VG	VB	A	I
8	B	VB	B	VL	71	VG	B	B	I
9	B	B	VB	VL	72	VG	A	VB	I
10	A	VB	VB	VL	73	VB	G	VG	FH
11	VB	VB	G	L	74	VB	VG	G	FH
12	VB	VB	VG	L	75	VB	VG	VG	FH
13	VB	B	A	L	76	B	A	VG	FH
14	VB	B	G	L	77	B	G	G	FH
15	VB	A	B	L	78	B	G	VG	FH
16	VB	A	A	L	79	B	VG	A	FH
17	VB	G	VB	L	80	B	VG	G	FH
18	VB	G	B	L	81	A	B	VG	FH
19	VB	VG	VB	L	82	A	A	G	FH
20	B	VB	A	L	83	A	A	VG	FH
21	B	VB	G	L	84	A	G	A	FH
22	B	B	B	L	85	A	G	G	FH
23	B	B	A	L	86	A	VG	B	FH
24	B	A	VB	L	87	A	VG	A	FH
25	B	A	B	L	88	G	VB	VG	FH
26	B	G	VB	L	89	G	B	G	FH
27	A	VB	B	L	90	G	B	VG	FH
28	A	VB	A	L	91	G	A	A	FH
29	A	B	VB	L	92	G	A	G	FH
30	A	B	B	L	93	G	G	B	FH
31	A	A	VB	L	94	G	G	A	FH
32	G	VB	VB	L	95	G	VG	VB	FH
33	G	VB	B	L	96	G	VG	B	FH
34	G	B	VB	L	97	VG	VB	G	FH
35	VG	VB	VB	L	98	VG	VB	VG	FH
36	VB	B	VG	FL	99	VG	B	A	FH
37	VB	A	G	FL	100	VG	B	G	FH
38	VB	G	A	FL	101	VG	A	B	FH
39	VB	VG	B	FL	102	VG	A	A	FH
40	B	VB	VG	FL	103	VG	G	VB	FH
41	B	B	G	FL	104	VG	G	B	FH
42	B	A	A	FL	105	VG	VG	VB	FH
43	B	G	B	FL	106	B	VG	VG	H
44	B	VG	VB	FL	107	A	G	VG	H
45	A	VB	G	FL	108	A	VG	G	H
46	A	B	A	FL	109	G	A	VG	H
47	A	A	B	FL	110	G	G	G	H
48	A	G	VB	FL	111	G	VG	A	H
49	G	VB	A	FL	112	VG	B	VG	H
50	G	B	B	FL	113	VG	A	G	H
51	G	A	VB	FL	114	VG	G	A	H
52	VG	VB	B	FL	115	VG	VG	B	H
53	VG	B	VB	FL	116	A	VG	VG	VH
54	VB	A	VG	I	117	G	G	VG	VH
55	VB	G	G	I	118	G	VG	G	VH
56	VB	VG	A	I	119	VG	A	VG	VH
57	B	B	VG	I	120	VG	G	G	VH
58	B	A	G	I	121	VG	VG	A	VH
59	B	G	A	I	122	G	VG	VG	EH
60	B	VG	B	I	123	VG	G	VG	EH
61	A	VB	VG	I	124	VG	VG	G	EH
62	A	B	G	I	125	VG	VG	VG	EH
63	A	A	A	I					

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